Two-Sample Tests of Hypothesis

Chapter 11

Learning Objectives

LO11-1 Test a hypothesis that two independent population means are equal, assuming that the population standard deviations are known and equal

LO11-2 Test a hypothesis that two independent population means are equal, with unknown population standard deviations

LO11-3 Test a hypothesis about the mean population difference between paired or dependent observations

LO11-4 Explain the difference between dependent and independent samples

Comparing Two Population Means

- In comparing two populations, we wish to know whether their means could be equal
- We are investigating whether the distribution of the difference between the means could have a mean of 0

- Examples
- Is there a difference in the mean value of residential real estate sold by male agents and female agents in south Florida
- Is there an increase in the production rate after music is piped into the production area

Comparing Two Population Means

- We can use the following formula to compute z if the following conditions are met
  - The two populations follow normal distributions
  - The samples are from independent (unrelated) populations
  - The population standard deviations are known

TWO-SAMPLE TEST OF MEANS—KNOWN σ

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad \textbf{(11–2)}$$

  - In the z formula, $\bar{x}_1 - \bar{x}_2$, is the difference in the sample means and the square root of the variance found with formula 11-1 is the standard deviation

Comparing Two Population Means Example

Customers at the FoodTown Supermarket have a choice when paying for their groceries. They may check out and pay using the standard cashier-assisted checkout or they may use the new Fast Lane procedure (self-checkout). The store manager would like to know if the mean checkout time using the standard checkout method is longer than using the Fast Lane. The time was measured from when the customer enters the line until all his or her bags are in the cart.

Step 1: State the null and alternate hypothesis

$$H_0: \mu_S \leq \mu F$$

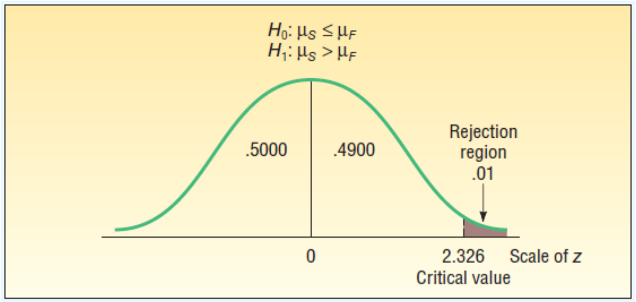
$$H_1: \mu_S > \mu_F$$

Step 2: Select the level of significance, we decide to use .01

Step 3: Determine the test statistic, we'll use z

Comparing Two Population Means Example Continued

Step 4: Formulate the decision rule, Reject H_0 if $z > 2.326$



Customer Type	Sample Mean	Population Standard Deviation	Sample Size
Standard	5.50 minutes	0.40 minute	50
Fast Lane	5.30 minutes	0.30 minute	100

Step 5: Make the decision regarding H_0, FoodTown randomly selected 50 customers using the standard checkout and computed a mean time of 5.5 minutes and randomly selected 100 customers using the Fast Lane and computed a mean time of 5.3 minutes. We will reject the null hypothesis.

$$z = \frac{\bar{x}_S - \bar{x}_F}{\sqrt{\frac{\sigma_S^2}{n_S} + \frac{\sigma_F^2}{n_F}}} = \frac{5.5 - 5.3}{\sqrt{\frac{0.40^2}{50} + \frac{0.30^2}{100}}} = \frac{0.2}{0.064031} = 3.123$$

Step 6: Interpret the result, the difference of .20 minute is too large to have occurred by chance. We conclude the Fast Lane method is faster.

Compare Two Means Using t

- There are two major differences in this test and the test just described in this chapter
  - We assume the sampled populations have equal but unknown standard deviations
  - We use the t distribution
- The three requirements for the test
  - The sampled populations are approximately normally distributed
  - The sampled populations are independent
  - The standard deviations of the two populations are equal

Compare Two Means Using t

- Finding the value of t requires two steps
- The first step is to pool the standard deviations according to the following formula

POOLED VARIANCE

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \quad (11\text{–}3)$$

- The value of t is computed from the following formula

TWO-SAMPLE TEST OF MEANS—UNKNOWN σ'S

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (11\text{–}4)$$

- The degrees of freedom for the test are $n_1 + n_2 - 2$

Two-Sample Pooled Test Example

Owens Lawn Care Inc. manufactures and assembles lawnmowers that are shipped to dealers throughout the United States and Canada. Two different procedures have been proposed for mounting the engine on the frame of the lawnmower, the Welles method and the Atkins method. The question is, is there a difference in the methods' mean time to mount the engines on the frames of the lawnmowers?
A time and motion study is conducted to evaluate.

Step 1: State the null and alternate hypothesis

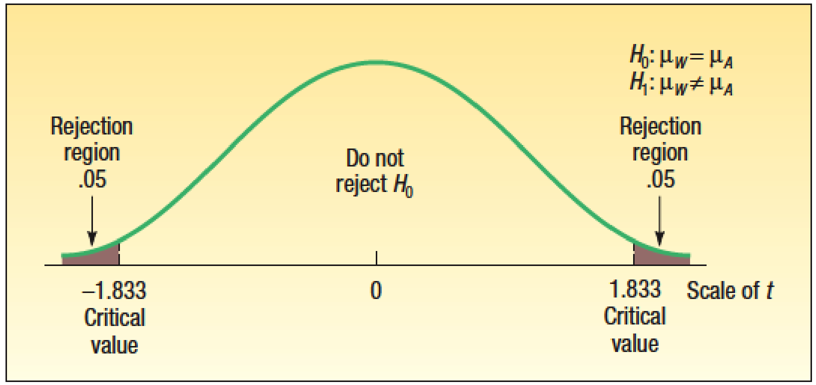
$$H_0: \mu_W = \mu A$$
$$H_1: \mu_W \neq \mu_A$$

Step 2: Select the level of significance, we decide to use .10
Step 3: Determine the test statistic, we'll use t

Welles (minutes)	Atkins (minutes)
2	3
4	7
9	5
3	8
2	4
	3

Two-Sample Pooled Test Example Continued

Step 4: Formulate the decision rule, do not reject H_0 if t falls between -1.833 and 1.833



Step 5: Make decision regarding H_0.
It takes three steps to compute the value of t.
First:
Calculate the sample standard deviations

Welles Method		Atkins Method	
x_W	$(x_W - \bar{x}_W)^2$	x_A	$(x_A - \bar{x}_A)^2$
2	$(2-4)^2 = 4$	3	$(3-5)^2 = 4$
4	$(4-4)^2 = 0$	7	$(7-5)^2 = 4$
9	$(9-4)^2 = 25$	5	$(5-5)^2 = 0$
3	$(3-4)^2 = 1$	8	$(8-5)^2 = 9$
2	$(2-4)^2 = 4$	4	$(4-5)^2 = 1$
20	34	3	$(3-5)^2 = 4$
		30	22

$$\bar{x}_W = \frac{\Sigma x_W}{n_W} = \frac{20}{5} = 4 \qquad \bar{x}_A = \frac{\Sigma x_A}{n_A} = \frac{30}{6} = 5$$

$$s_W = \sqrt{\frac{\Sigma(x_W - \bar{x}_W)^2}{n_W - 1}} = \sqrt{\frac{34}{5-1}} = 2.9155 \qquad s_A = \sqrt{\frac{\Sigma(x_A - \bar{x}_A)^2}{n_A - 1}} = \sqrt{\frac{22}{6-1}} = 2.0976$$

Two-Sample Pooled Test Example Concluded

Second: Pool the sample variances

$$s_p^2 = \frac{(n_W - 1)s_W^2 + (n_A - 1)s_A^2}{n_W + n_A - 2} = \frac{(5 - 1)(2.9155)^2 + (6 - 1)(2.0976)^2}{5 + 6 - 2} = 6.2222$$

Third: Determine the value of t

$$t = \frac{\bar{x}_W - \bar{x}_A}{\sqrt{s_p^2\left(\frac{1}{n_W} + \frac{1}{n_A}\right)}} = \frac{4.00 - 5.00}{\sqrt{6.2222\left(\frac{1}{5} + \frac{1}{6}\right)}} = -0.662$$

The decision is not to reject the null hypothesis because –0.662 falls in the region between -1.833 and 1.833.

Step 6: Interpret the result, we conclude the sample data failed to show a difference between the mean assembly times of the two methods.

Unequal Population Standard Deviations

- If we cannot assume the population standard deviations are equal, we adjust the degrees of freedom and the formula for finding t
- We determine the degrees of freedom based on the following formula

DEGREES OF FREEDOM FOR UNEQUAL VARIANCE TEST

$$df = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} \qquad [11\text{–}6]$$

- The value of the test statistic is computed from the following formula

TEST STATISTIC FOR NO DIFFERENCE IN MEANS, UNEQUAL VARIANCES

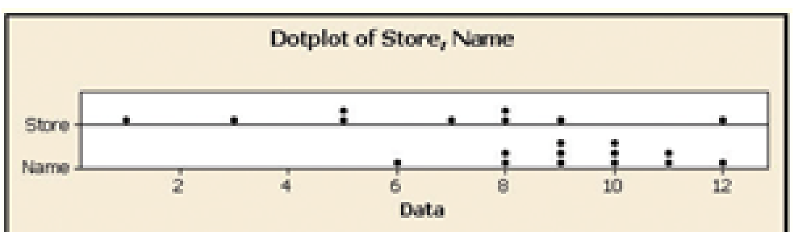
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad [11\text{–}5]$$

Unequal Population Standard Deviations Example

Personnel in a consumer testing laboratory are evaluating the absorbency of paper towels. They wish to compare a set of store brand towels to a similar group of name brand towels. For each brand, they dip a ply of the paper into a tub of fluid, allow the paper to drain back into the vat for 2 minutes, and then evaluate the amount of liquid the paper has taken up from the vat.

A random sample of 9 store brand towels absorption amounts (in ml.)								
8	8	3	1	9	7	5	5	12

A random sample of 12 name brand towels absorption amounts (in ml.)											
12	11	10	6	8	9	9	10	11	9	8	10



Descriptive Statistics: Store, Name

Variable	N	Mean	StDev
Store	9	6.44	3.32
Name	12	9.417	1.621

Unequal Population Standard Deviations Example Continued

Step 1: State the null and alternate hypothesis

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$

Step 2: Select the level of significance, we decide to use .10

Step 3: Determine the test statistic, we'll use t

We must adjust the degrees of freedom with formula 11-6 before finding the critical values and round the result down to an integer; in this case, 10

$$df = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} = \frac{[(3.321^2/9) + (1.621^2/12)]^2}{\frac{(3.321^2/9)^2}{9 - 1} + \frac{(1.621^2/12)^2}{12 - 1}} = \frac{1.4444^2}{.1877 + .0044} = 10.86$$

Step 4: State the decision rule, do not reject H_0 if t falls between -1.812 and 1.812.

Step 5: Make decision, we reject the null hypothesis.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{6.444 - 9.417}{\sqrt{\frac{3.321^2}{9} + \frac{1.621^2}{12}}} = -2.474$$

Step 6: Interpret, the mean absorption rate of the two types of towels is not the same.

Dependent Samples

- We first compute the mean and the standard deviation of the sample differences
- The value of the test statistic is computed with the following formula

PAIRED *t* TEST	$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$	(11–7)

  - There are n – 1 degrees of freedom
  - $\bar{d}$ is the mean of the difference between the paired observations
  - s_d is the standard deviation of the differences between the paired observations
  - n is the number of paired observations

Dependent Samples Continued

- Note: the standard deviation of the differences will be computed with the formula 3-11, except d is substituted for x

$$s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$$

- Example
- Nickel Savings and Loan employs two firms, Schadek Appraisals and Bowyer Real Estate to appraise the value of the real estate on which it makes loans. To review the consistency of the two appraisal firms, Nickel randomly selects 10 homes and has both of the firms appraise the values of the selected homes. Thus, there will be a pair of values for each home, these appraised values are related to the home selected. This is called a paired sample.

Dependent Samples Example

Recall that Nickel Savings and Loan wishes to compare the two companies it uses to appraise the value of residential homes. Nickel Savings selected a sample of 10 residential properties and scheduled both firms for an appraisal. The results are reported in $000. At the .05 significance level, can we conclude there is a difference between the firm's appraised values?

Home	Schadek	Bowyer
1	235	228
2	210	205
3	231	219
4	242	240
5	205	198
6	230	223
7	231	227
8	210	215
9	225	222
10	249	245

Step 1: State the null and alternate hypothesis

$$H_0: \mu_d = 0$$
$$H_1: \mu_d \neq 0$$

Step 2: Select the level of significance, we decide to use .05

Step 3: Determine the test statistic, we'll use t

Step 4: State the decision rule, reject H_0 if $t < -2.262$ or > 2.262

Dependent Samples Example Continued

Home	Schadek	Bowyer	Difference, d	$(d-\bar{d})$	$(d-\bar{d})^2$
1	235	228	7	2.4	5.76
2	210	205	5	0.4	0.16
3	231	219	12	7.4	54.76
4	242	240	2	−2.6	6.76
5	205	198	7	2.4	5.76
6	230	223	7	2.4	5.76
7	231	227	4	−0.6	0.36
8	210	215	−5	−9.6	92.16
9	225	222	3	−1.6	2.56
10	249	245	4	−0.6	0.36
			46	0	174.40

$$\bar{d} = \frac{\Sigma d}{n} = \frac{46}{10} = 4.60$$

$$s_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}} = \sqrt{\frac{174.4}{10-1}} = 4.402$$

Using formula (11–7), the value of the test statistic is 3.305, found by

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{4.6}{4.402/\sqrt{10}} = \frac{4.6}{1.3920} = 3.305$$

Here we find the mean of the sample differences, $\bar{d}$ is 4.6 and the standard deviation of the sample differences, s_d is 4.402. use these in formula 11-7 to compute the t value, 3.305

Step 5: Make your decision, we'll reject the null hypothesis

Step 6: Interpret, we conclude there is a difference between the firms' mean appraised home values

Dependent and Independent Samples

- There are two types of dependent samples
- Those characterized by a measurement, an intervention of some type and then another measurement
  - For example, suppose we wish to show that by playing music in the production area we are able to increase production. We begin by selecting a sample of workers and measure their output, then we place the speakers in the production area and play soothing music, and then we again measure the output
- A matching or pairing of the observations
  - For example, the Nickel Savings and Loan example illustrates dependent samples because a property is selected and both firms appraise the same property
- We prefer a test based on dependent samples because it reduces the amount of variation in the test and is considered a better test